A TECHNIQUE TO COMPARE EWMAD2 SCHEME WITH DIFFERENT SAMPLE SIZES

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Abstract

Basically the Exponentially Weighted Moving Average (EWMA) charts were introduced for monitoring the process mean in 1959 by Robert. It is much sensitive in detecting small shifts of mean than Shewhart charts. But lately it was felt that monitoring process mean was not enough in some cases. Then it was essential to introduce EWMA chart for monitoring sample variance and such a chart was introduced by Chen and Gan in 1993. These both EWMA charts can be used to monitor process mean and variance independently. But later it was identified monitoring mean and variance is a bivariate problem. In this series Exponentially Weighted Moving Average Distance square scheme (EWMAD2) was introduced with the claim that the control limits of the schemes are independent of sample size. In this research project the effect of sample size in EWMAD2 scheme is analyzed with the simulated samples with the different sample size and Average Run Lengths. In conclusion it is proved that EWMAD2 scheme is independent of sample size in determining the control limits.

Keywords: Average run length (ARL), EWMA control chart, control limits (CL)

Introduction

EWMA charts

EWMA chart was developed by Robbert in 1959[13]. This control chart is much advance than Shewhart charts in detecting small shifts of the process. It used the most recent and past observations too. Hence it can be categorize in to memory control charts and it leads to have a high sensitivity same as CUSUM control charts. The major uses of EWMA charts are detecting shifts and forecasting the process.

EWMA charts for monitoring sample mean

EWMA chart was introduced for monitoring the sample mean of a quality parameter which is more sensitive even in small shifts. In EWMA chart, EWMA Variable A_t is plotted against the sample number t ($t = 1,2,..$), where

$$A_t = (1 - \lambda_m)A_{t-1} + \lambda_m \bar{X}_t. \tag{1}$$

Here A_0, is considered as the target mean μ_0, λ_m is a constant such that $0<\lambda_m< 1$. It is selected based on the shift in the mean to be detected quickly for any process for a given in-control average run length (ARL). $\bar{X}_t$ is the t^{th} sample mean of the quality parameter, to be monitored.

This chart issues an out-of-control signal if A_t is greater than the upper control limit (UCL) or lower than the lower control limit (LCL). For designing this chart, λ_m values and control limits for detecting different shifts in mean under different sample sizes and in-control ARLs, can be found in Crowder (1989) [4].

EWMA charts for monitoring sample variance

An EWMA chart for monitoring sample variance of a quality parameter was introduced by Chang and Gan in 1995 [6]. In this chart, the EWMA variable B_t is plotted against the sample number t ($t = 1,2,..$), where

$$B_t = (1 - \lambda_v)B_{t-1} + \lambda_v \log(S_t^2). \qquad (2)$$

Here the value of $B_t$is taken as E[log(S_t^2)] and $S_t^2$is the sample variance of the quality parameter interested. λ_v is a positive constant which having the possible values of $0 < \lambda_v < 1$ and it is based on the shift in the variance to be detected quickly for a given in-control ARL. Like in the EWMA chart for monitoring sample mean, this chart also issues an out-of-control signal if B_t is greater than UCL or the lower than the LCL

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EWMA charts for jointly monitoring sample mean and variance

The above discussed two EWMA charts are used for monitoring the process mean and variance independently. In 1997, it was understood that monitoring the sample mean and variance is a bivariate problem and these two has to be monitored jointly [7]. A joint monitoring scheme for monitoring mean and variance simultaneously using EWMA technique was introduced by Gan [6]. Another two joint monitoring scheme, called max EWMA scheme and EWMA semicircle scheme for joint monitoring of process mean and variance were proposed by Chen *et al.* [2] [3]. The scheme parameters to design these joint monitoring schemes differs based on the sample sizes. This issue restricts the users in selecting a convenient sample size.

Statistical Process Control

Statistical Process Control (SPC) is a statistical method of separating variation resulting from special causes from natural variation and to establish and maintain consistency in the process, enabling process improvement" (Goetsch& Davis, 2003). SPC is an important application in statistics and control charts are the most important tool in SPC. The major quality tools used I the industry are given below:

- Check Sheet
- Cause-and-Effect Sheet
- Flow Chart
- Pareto Chart
- Scatter Diagram
- Probability Plot
- Histogram
- Control Charts

These control charts was introduced in 1924. Shewhart$\bar{X}$ chart is considered as the most popular category to monitor the mean of a process. But Cumulative Sum and Exponential weighted moving average are good alternatives to Shewhart $\bar{X}$ chart those are specially designed for detecting small shifts in the process mean. Control charts can be divide in to two main categories. Those are memory less control charts and memory control charts. Shewhart type control charts can be categorize under memory less control charts since it ignores the past information and CUSUM and EWMA charts can be count as memory control charts. Generally every manufacturing process has its variations. But large variations can take the process mean far away from the target value. So the main purpose of a SPC is to give a signal when the process mean has moved away from the target mean and another purpose is to give a signal when the variability has increased.

Related researches

There are some related research done by various researchers previously. Lots of them were talked about EWMA charts. They were talked about EWMAD2 scheme very rarely. But these are some important related researches done previously.Razmy and Peiris (2013) have discussed the performance comparison of Shewhart joint monitoring schemes for mean and variance. Performance of four schemes (shewhart combined scheme with rectangular control region (SSr), Shewhart control scheme with elliptical control region (SSe), joint monitoring scheme (SSm) and Shewhart distance scheme (SSd)) were compared under a common flat form in this research and they have obtained tha the Shewhart distance scheme performs best and the Shewhart scheme with rectangular acceptance region performs in a poorest way.

Nasir Abbas *et.al.*, (2012) has presented a EWMA-type control chart for monitoring the process mean using auxiliary information. Here, they propose an EWMA- type control chart which utilizes a single auxiliary variable by targeting on small and moderate shifts in the process mean. So they show that the proposed chart is performing better than its univariate and bivariate competitors which are designed for detecting small shifts.

Sin Yin *et.al.,* has published a research paper in 2015 about a study on S^2 EWMA chart for monitoring process variance based on the MRL performances. It says existing design of S^2 EWMA control chart monitor the sample variance of a process based on Average Run Length (ARL) criterion. But it makes the changes in the shape of the run length distribution with the magnitude of shift in the variance. So they suggest MRL (median run length) instead of Average Run Length since it gives a more meaningful explanation about the in control and out of control performances in control charts.

Saddam AkberAbbasi (2010) also presented on sensitivity of EWMA control chart for monitoring process dispersion. Generally most of the Exponential Weighted Moving Average charts proposed so far based on asymptotic nature of control limits due to the easiness of computations. But here it has been shown that quick detection of initial out of control conditions can be achieved by using exact or time varying control limits. Furthermore the researcher says that the fast initial response (FIR) feature can increase the sensitivity of EWMA charts for detecting process shifts.

Kalgonda*et.al*., (2010) they have discussed about Exponentially Weighted Moving Average control charts in their research article. In this research they have carried out a simulation to calculate the average run length rules using C- programs. So they could observe that approximately same values as ARLs could obtain by simulation method by using C-programs. That is same as the ARL values in the Markov chain approach obtained by Lucas, Saccucci. Several examples had used in this research paper with numerical outputs and graphical representation to make the article success.

Methodology

A new EWMAD2 joint monitoring scheme was introduced with a claim that its control limits are independent of sample size [12]. This scheme uses the standardized sample mean U_t and variance V_t such that

$$U_t = \frac{\bar{X}_t - \mu_0}{\sigma_0 / \sqrt{n}} \qquad (3)$$

And

$$V_t = \Phi^{-1}\left[H\left(\frac{(n-1)S_t^2}{\sigma_0^2}:n-1\right)\right]. \quad (4)$$

$$H\left[\frac{(n-1)S_t^2}{\sigma_0^2}:n-1\right] \quad = H(w;v) = P(W \leq w)\ for\ W\sim\chi_v^2 \quad (5)$$

the chi-square distribution with v degrees of freedom and ϕ(.) is the cumulative distribution function of a standard normal random variable .S_t is the sample standard deviation, σ_0 is the population standard deviation and *n* is the sample size.
A statisticsD_t^2 is defined by

$$D_t^2 = U_t^2 + V_t^2. \quad (6)$$

The EWMAD2 scheme is obtained by plotting the EWMAD2 statistic C_t against the sample number *t* where

$$C_t = (1-\lambda_d)C_{t-1} + \lambda_d D_t^2. \quad (7)$$

C_0= $E(D_t^2)$ = 0 and λ_d is a constant selected based on the shift in D_t^2, to be detected quickly. The optimum λ_d values for detecting various shifts in D_t^2, for the selected in-control ARLs can be read in Razmy (2005) [12]. The sample sizes (*n*) studied were 5, 10, 50, 100 and 150. Samples were simulated in SAS using proc RANNO with sample sizes *n*. For each sample, the statistics D_t^2 was calculated from the variables u_t and v_t. The statics C_t was obtained for different values of $\lambda_d(\lambda_d = 0.05, 0.1, 0.2, \ldots., 0.9, 1.0)$.

Initially arbitrary control limits (*CL*) were assumed for each combination of λ_d and in-control ARLs. Then, if C_t<*CL*, then the second sample was simulated. This procedure was continued till C_t>*CL* where C_t is an out-of-control point. Each time, the number of samples generated till to find an out-of-control point was recorded and it is the run length.

In the same way 100,000 runs were performed and the ARLs were found. Subsequently, the assumed *CL* values were adjusted till to obtain the required in-control ARLS of 100, 250, 300, 370, 500 and 1000 for different λ_d values. By this procedure, the *CL*s for different combinations of in-control ARLs, sample sizes and λ_d values were found. In all cases, simulations were run until the standard error of the ARL was less than 1% of the pre-specified ARL. The written programs are given in the Appendix. The obtained *CL*s were plotted against the lambda value for the selected in-control ARLS and sample sizes.

Results and Discussion

Tables 4.1 to 4.5 shows the *CL*s for different combination of in-control ARLs and sample size. Further, obtained *CL*s were plotted against the lambda value for the selected in-control ARLS and sample sizes in figures 4.1 to 4.5.

From the figures, it could be observed the control limits is increases withλ_d. It is obvious the *CL* increase with in-control ARLs for a given $\lambda_d$value.

Table 4.1: Control limits for selected in-control ARLS, n = 5

Λ	ARL=100	ARL=250	ARL=300	ARL=370	ARL=500	ARL=1000
0.05	2.4617	2.682	2.7231	2.7691	2.8339	2.9709

0.1	2.8902	3.21	3.268	3.3351	3.428	3.6345
0.2	3.6502	4.1281	4.2225	4.3243	4.4725	4.8035
0.3	4.3422	4.99	5.1172	5.2622	5.4575	5.9121
0.4	5.0255	5.8384	5.9894	6.1675	6.43	7.0073
0.5	5.6991	6.6848	6.8624	7.0876	7.3985	8.1095
0.6	6.3964	7.5297	7.7542	8.0156	8.3771	9.2192
0.7	7.0856	8.3868	8.645	8.9501	9.3685	10.3506
0.8	7.7706	9.2555	9.5552	9.894	10.3807	11.4901
0.9	8.4776	10.1502	10.4758	10.8453	11.391	12.6355
1	9.221	11.045	11.411	11.8332	12.435	13.8132

Table 4.2: Control limits for selected in-control ARLS, n = 10

Λ	ARL=100	ARL=250	ARL=300	ARL=370	ARL=500	ARL=1000
0.05	2.4598	2.6828	2.7238	2.77	2.8338	2.9714
0.1	2.8915	3.2085	3.2678	3.3336	3.4278	3.6326
0.2	3.6458	4.1258	4.2215	4.3288	4.4738	4.8008
0.3	4.3458	4.9901	5.113	5.2617	5.4578	5.9098
0.4	5.0385	5.8391	5.9931	6.1697	6.4303	7.0088
0.5	5.7158	6.6821	6.8731	7.088	7.398	8.1121
0.6	6.3942	7.5321	7.7551	8.0105	8.3771	9.2225
0.7	7.0855	8.3886	8.6484	8.9505	9.3614	10.343
0.8	7.7742	9.2486	9.5574	9.8861	10.3793	11.4865
0.9	8.5005	10.1453	10.4698	10.8451	11.385	12.6482
1	9.1912	11.0403	11.4128	11.8301	12.4328	13.8128

Table 4.3: Control limits for selected in-control ARLS, n = 50

Λ	ARL=100	ARL=250	ARL=300	ARL=370	ARL=500	ARL=1000
0.05	2.4628	2.6801	2.7219	2.7692	2.8326	2.9705
0.1	2.8928	3.2076	3.2681	3.3352	3.4286	3.6322
0.2	3.6485	4.1295	4.2181	4.3236	4.4731	4.8002
0.5	5.7063	6.6747	6.8686	7.0879	7.3981	8.1112
0.6	6.3951	7.5312	7.7502	8.0101	8.3715	9.2146
0.7	7.0851	8.3872	8.6465	8.9352	9.3611	10.3458
0.8	7.7725	9.2581	9.5565	9.8901	10.3653	11.4815
0.9	8.4952	10.1455	10.4721	10.8451	11.3852	12.6459
1	9.2103	11.0441	11.4121	11.8176	12.4331	13.8036

Table 4.4: Control limits for selected in-control ARLS, n = 100

Λ	ARL=100	ARL=250	ARL=300	ARL=370	ARL=500	ARL=1000
0.05	2.4613	2.6809	2.7235	2.7695	2.8311	2.9705
0.1	2.8931	3.2071	3.2668	3.3354	3.4274	3.6295
0.2	3.6487	4.1293	4.2199	4.3239	4.4718	4.8004
0.3	4.3435	4.9903	5.1146	5.2548	5.4552	5.9074
0.4	5.0303	5.8327	5.9902	6.1708	6.4271	7.0059
0.5	5.7065	6.6778	6.8685	7.0866	7.3962	8.1073
0.6	6.3949	7.5285	7.7502	8.0105	8.3734	9.2145
0.7	7.0852	8.3871	8.6466	8.9363	9.3654	10.3432
0.8	7.7728	9.2579	9.5559	9.8903	10.3753	11.4835
0.9	8.4955	10.1465	10.4734	10.8462	11.3972	12.6391
1	9.2105	11.0442	11.4123	11.8172	12.4201	13.8138

Table 4.5: Control limits for selected in-control ARLS, n = 150

Λ	ARL=100	ARL=250	ARL=300	ARL=370	ARL=500	ARL=1000
0.05	2.4618	2.6811	2.7215	2.7699	2.8342	2.9695

0.1	2.8927	3.2092	3.2662	3.3342	3.4268	3.6328
0.2	3.6442	4.1274	4.2185	4.3222	4.4701	4.7983
0.3	4.3452	4.9884	5.1145	5.2562	5.4522	5.9065
0.4	5.0305	5.8345	5.9902	6.1697	6.425	7.0071
0.5	5.7064	6.6761	6.8684	7.0842	7.3901	8.1035
0.6	6.395	7.5287	7.7492	8.0102	8.3721	9.2145
0.7	7.0853	8.3873	8.6465	8.9398	9.3645	10.3412
0.8	7.7756	9.2542	9.5553	9.8812	10.3732	11.4965
0.9	8.4951	10.1468	10.4712	10.8485	11.3897	12.6352
1	9.2101	11.0362	11.4102	11.8264	12.4262	13.8235

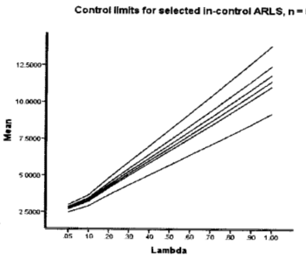

Figure 4.1: Control limits for selected in-control ARLS, n = 5

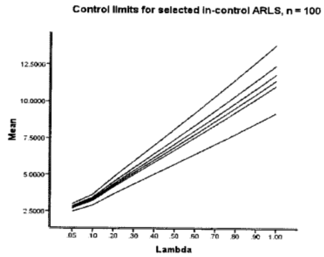

Figure 4.4: Control limits for selected in-control ARLS, n = 100

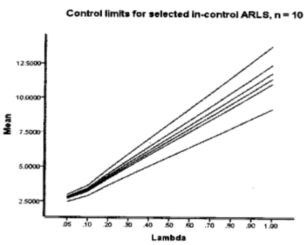

Figure 4.2: Control limits for selected in-control ARLS, n = 10

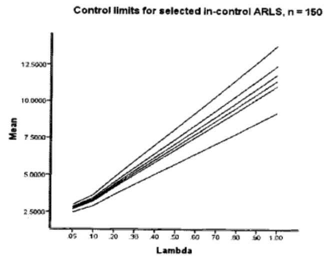

Figure 4.5: Control limits for selected in-control ARLS, n = 150

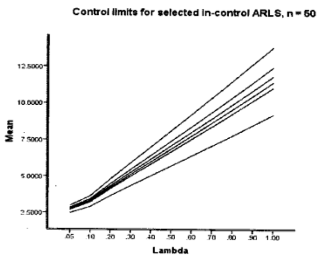

Figure 4.3: Control limits for selected in-control ARLS, n = 50

Conclusions

Size of the sample is to be selected is very important case in quality monitoring. For some cases the final result can dependent on the sample size that has been selected. In this research project it has considered a technique to compare EWMAD2 scheme with different sample sizes. According to the results it can be concluded that CLs are independent of sample size. So any size of the sample size will obtain the certain CLs with corresponding ARLs. Also apart from that it could be observed the control limits increase with $\lambda_{d.}$ This property of independent of sample size in EWMAD2 scheme is a very important characteristic that ease the procedure of designing this schemes with required sample sizes.

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